Error Minimization of Averaged G2

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Supose we have N data points: $x_i \pm \sigma_i$ for $i = 1$ to N. We would like to take a weighted aveage $\bar{x} = \sum \alpha_i x_i$ with $\sum \alpha_i = 1$. We will adjust α_i so that we can minimize the uncertainty in the average. The combined uncertainty is given by

$$\bar{\sigma} = \sqrt{\sum \alpha_i^2 \sigma_i^2}. \tag{1}$$

We use Lagrange multiplier to minimize

$$f\left(\alpha_1, \alpha_2, \cdots, \alpha_N\right) = \sum \alpha_i^2 \sigma_i^2, \tag{2}$$

subject to

$$g\left(\alpha_1, \alpha_2, \cdots, \alpha_N\right) = \sum \alpha_i = 1. \tag{3}$$

We then construct an auxiliary function

$$\Lambda\left(\alpha_1, \alpha_2, \cdots, \alpha_N, \lambda\right) = f + \lambda\left(g - 1\right). \tag{4}$$

Solving

$$\nabla_{\alpha_i, \lambda} \Lambda = 0 \tag{5}$$

for the weights α_i, we have

$$\frac{\partial \Lambda}{\partial \alpha_i} = 2\alpha_i \sigma_i^2 + \lambda = 0 \tag{6}$$

and

$$\frac{\partial \Lambda}{\partial \lambda} = \sum \alpha_i - 1 = 0. \tag{7}$$

Therefore, we have

$$\lambda = \frac{-2}{\sum \frac{1}{\sigma_i^2}} \tag{8}$$

and

$$\alpha_i = \frac{-\lambda}{2\sigma_i^2} = \frac{1}{\sigma_i^2 \sum \frac{1}{\sigma_i^2}} \tag{9}$$